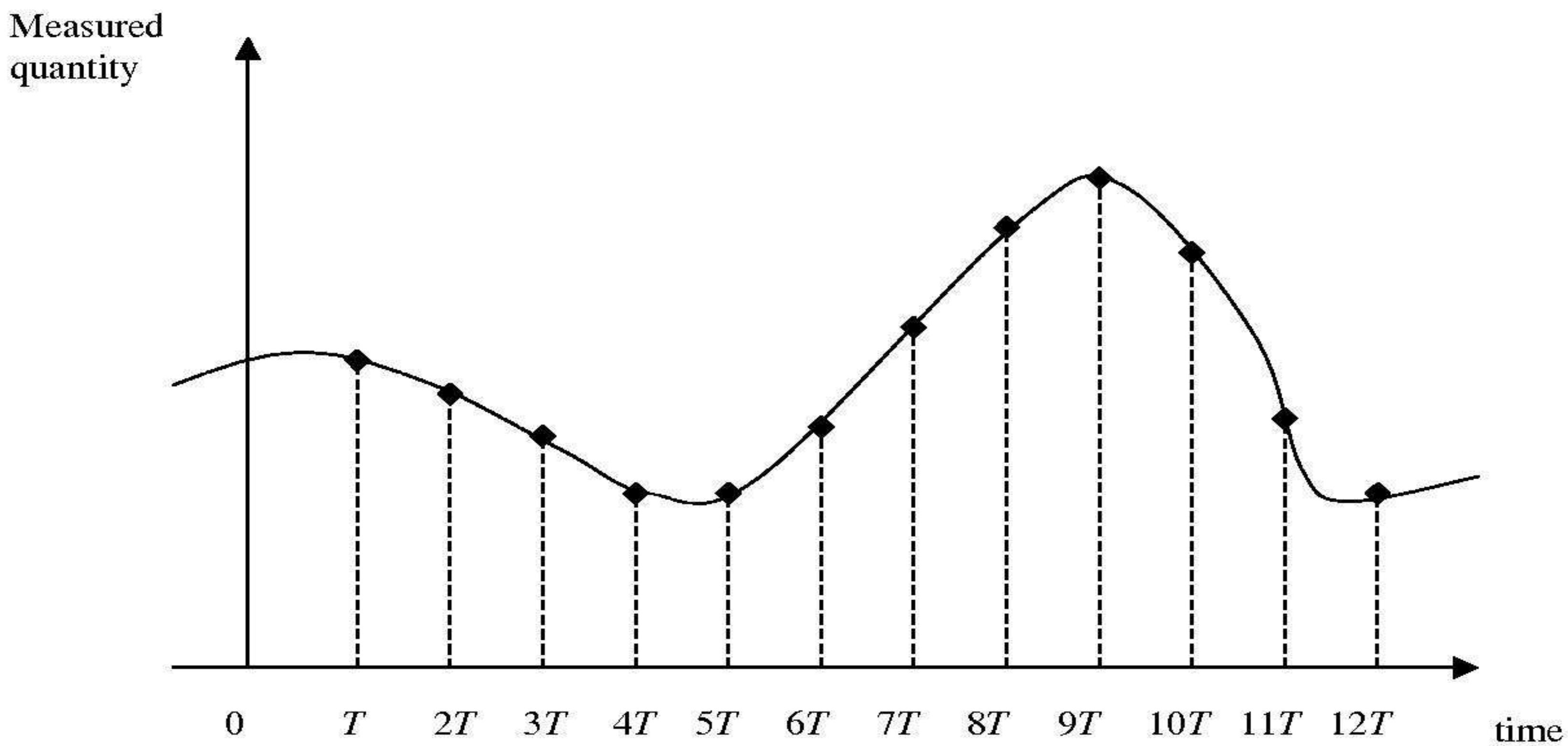
First steps in digital signal processing

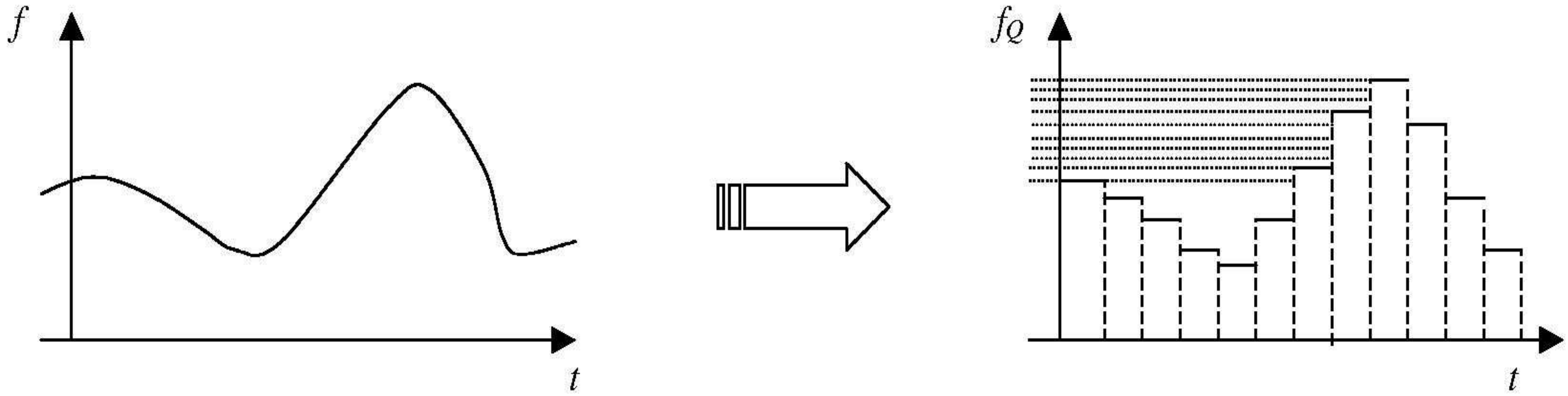
John Coleman

Phonetics Laboratory
University of Oxford

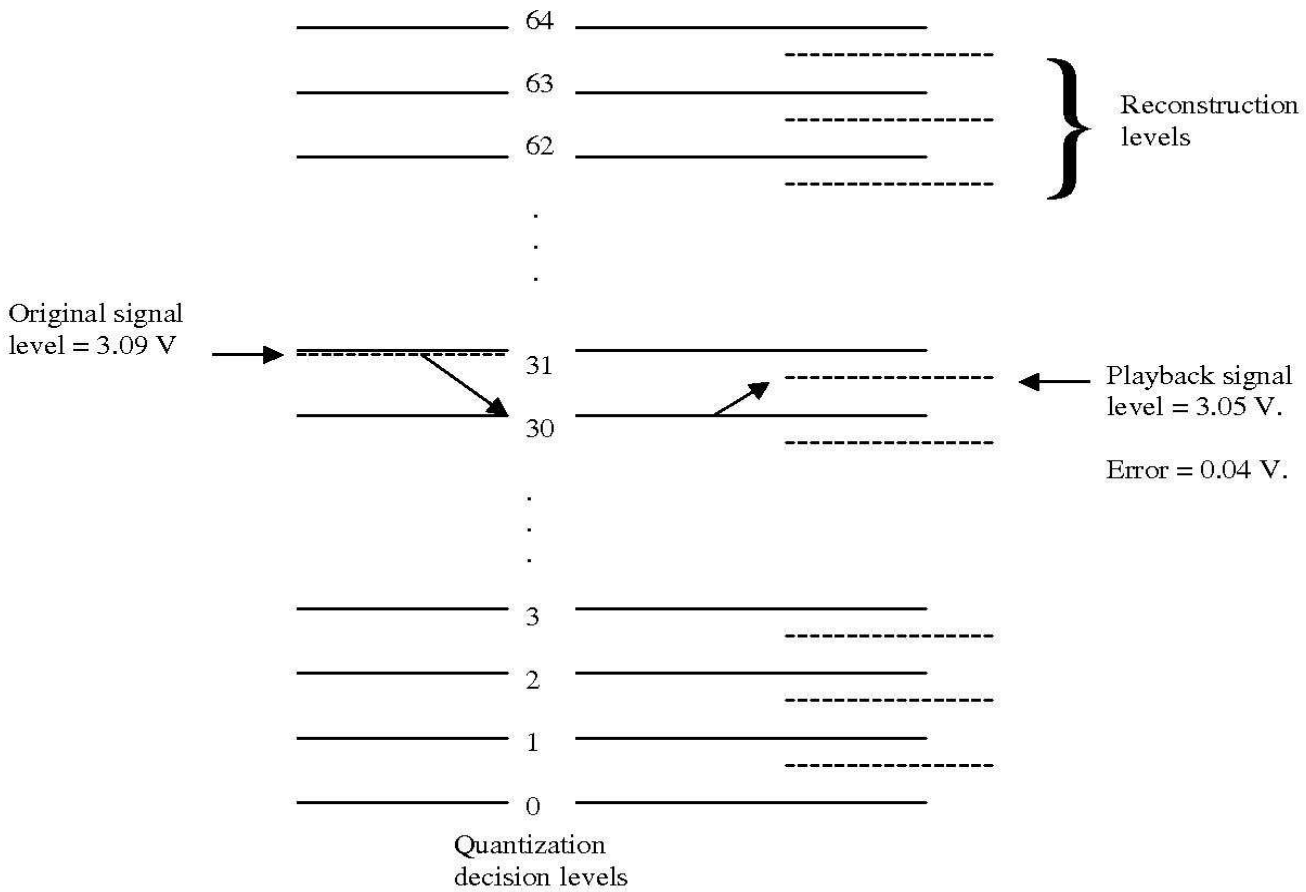
Analogue-to-digital conversion 1: Sampling



Analogue-to-digital conversion 2: Quantization



D-to-A, quantization error

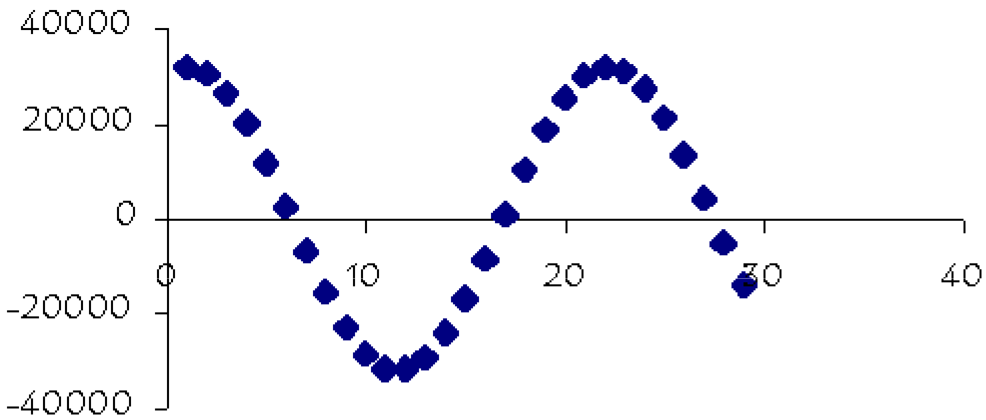


D-to-A, quantization error

- To reduce the quantization error, use more levels (dynamic range):

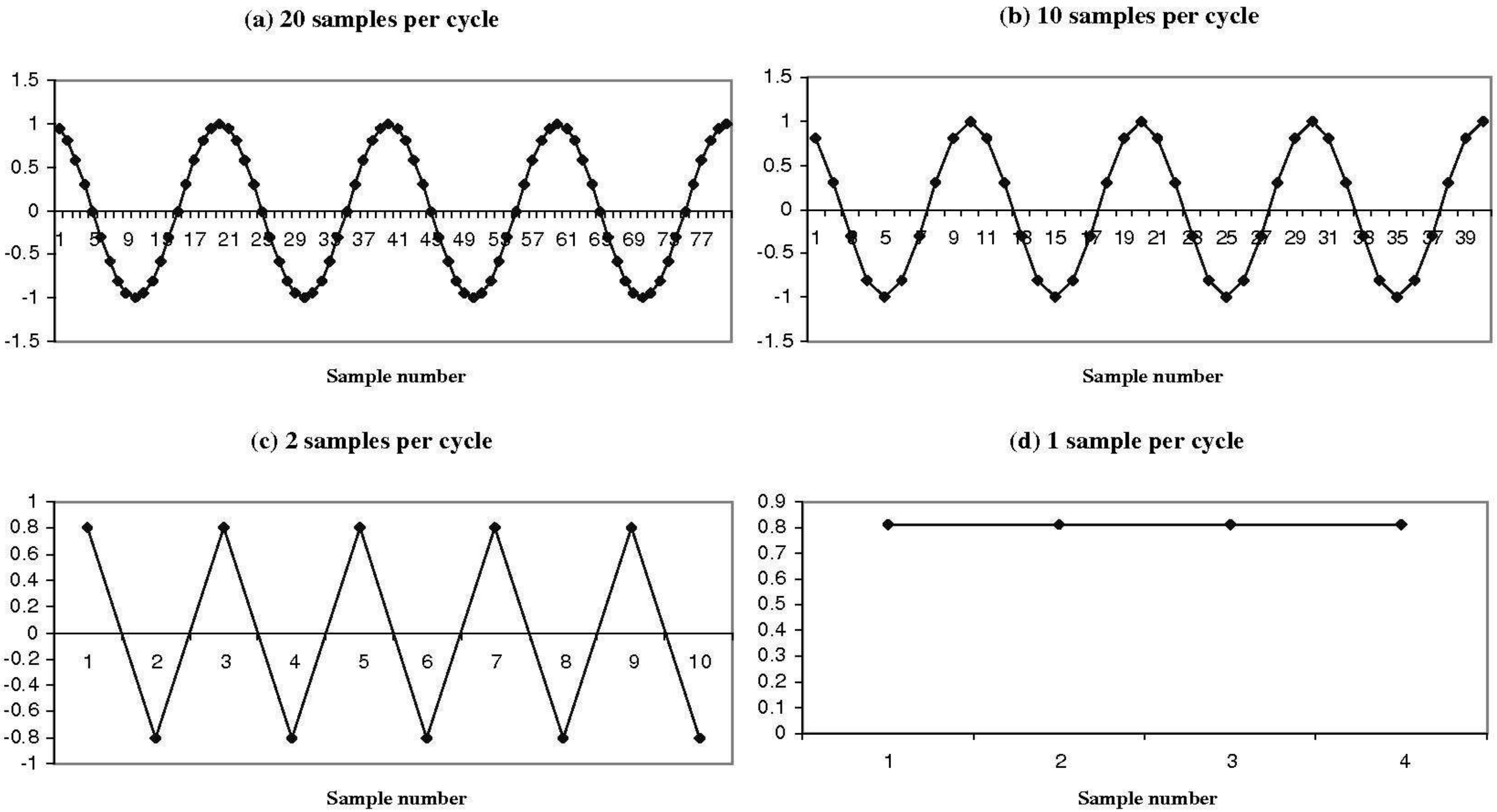
- 8 bits: $2^8 = 256$ levels
- 12 bits: $2^{12} = 4096$ levels
- 16 bits: $2^{16} = 65536$ levels

Pulse Code Modulation (Alec Reeves 1937)



Sampling theorem; Nyquist frequency

- Sampling rate must be at least twice the highest frequency you want to capture



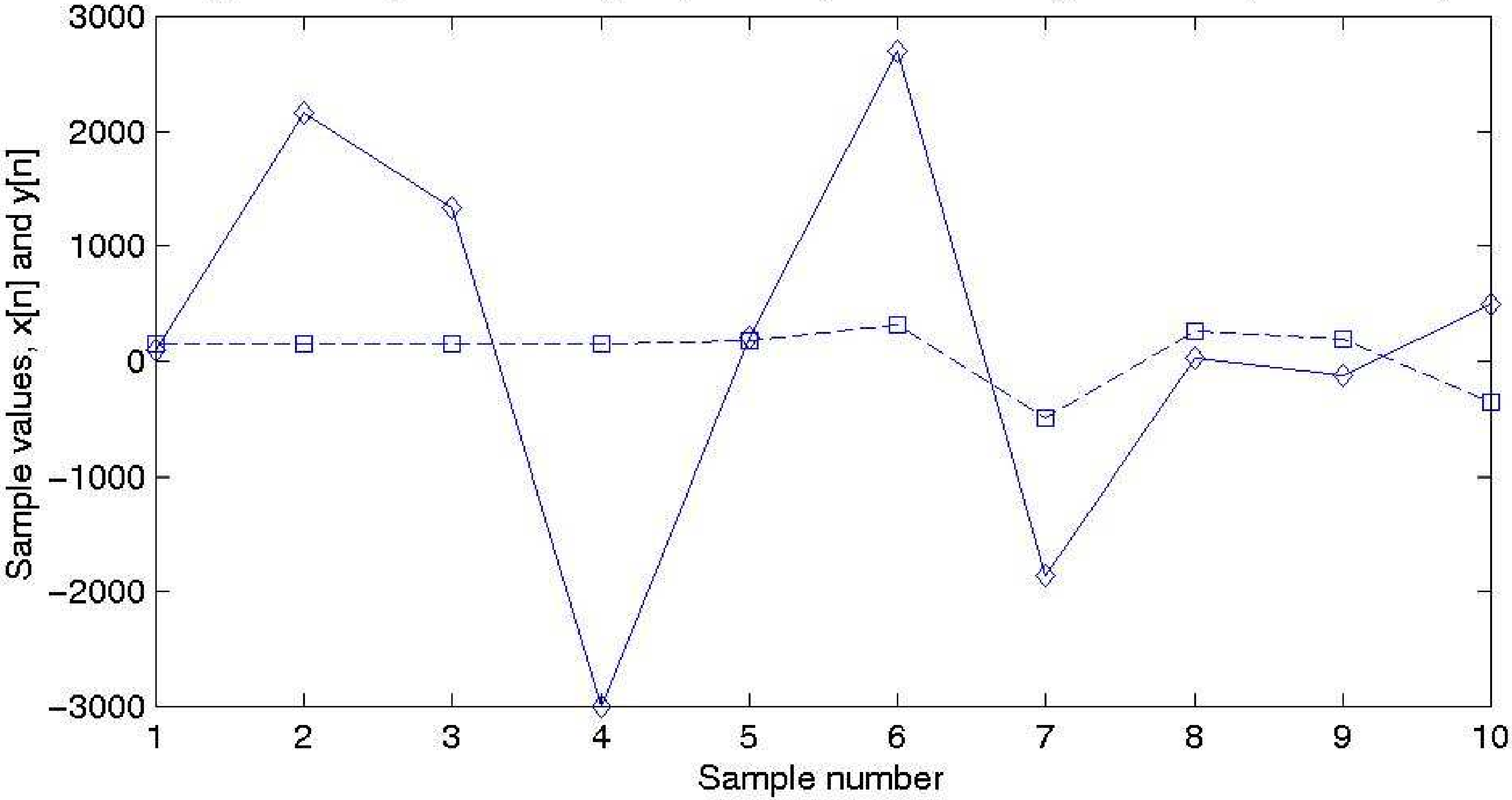
Operations on sequences of numbers

- Let's call the sample number i and the i'th sample x[i]
- **Sum** or **integral**, $\Sigma x[i]$.
- If x[i] has positive and negative values, take $|x[i]|$ the absolute (i.e. unsigned) value of x[i].
- Or, first calculate the square of x[i], $x[i]^2$, and then take the square root, $\sqrt{(x[i]^2)}$. $\Sigma\sqrt{(x[i]^2)}$ is a measure of the overall energy of a signal.
- $x[i]^2$ gets bigger and bigger as x[i] gets longer.
- The *average* amplitude of a signal, calculated over n samples: $\sqrt{(\Sigma x[i]^2/n)}$. This is called the *root mean square* or RMS amplitude.

Local (moving) average

e.g., $y[n] = \frac{1}{4} x[n] + \frac{1}{4} x[n-1] + \frac{1}{4} x[n-2] + \frac{1}{4} x[n-3]$

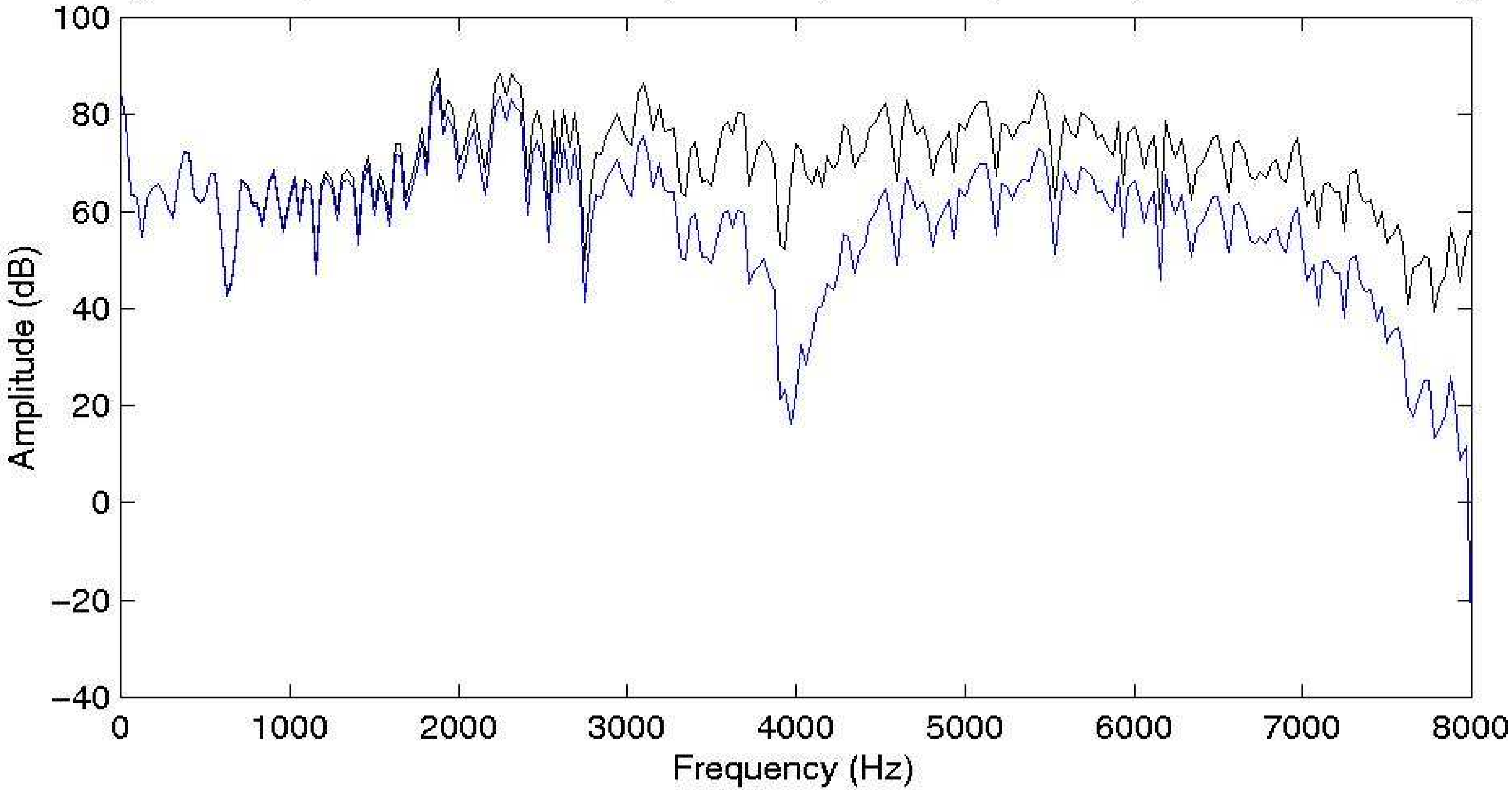
Figure 3.1. A portion of a signal (solid line) and its running mean of 4 (dashed line).



Local (moving) average

Effect: low-pass filtering

Figure 3.2. Spectrum of "sh" before (black line) and after (blue line) means–of–4 filtering.



Time-domain filtering

4 samples is very short, so its effects are very local – high frequency components. To smooth over a larger slice of the signal, we can do two things:

a) increase the number of samples in x[n] … x [n-m]

b) make y[n] depend in part on its own previous value, y[n–1], or several previous values.

Time-domain filtering

- A filter of the second kind has the general form:

$$y[n] = b_0 x[n] + b_1\, x[n-1] + b_2\, x[n-1] + \ldots + b_k\, x[n-k] \\ - a_1\, y[n-1] + \ldots. + a_j\, y[n-j]$$

- By varying the a's, b's, and the number and spacing of previous x and y samples, a variety of filters with various kinds of frequency-selecting behaviours can be constructed.

Linear Prediction (preview)

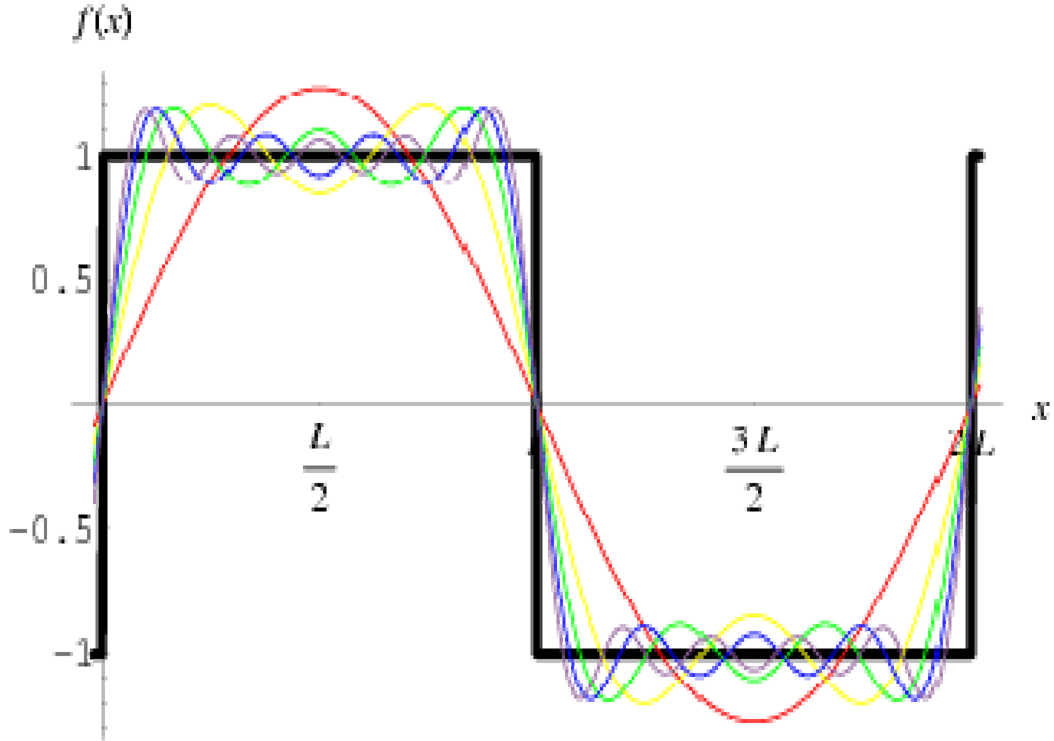
- $y[n] = b_0 x[n] + b_1\, x[n-1] + b_2\, x[n-1] + \ldots + b_k\, x[n-k]$
  $- a_1\, y[n-1] + \ldots. + a_j\, y[n-j]$

- We can estimate the magnitude of the current sample as a linear combination of the previous p samples (typically 12 to 18 samples):

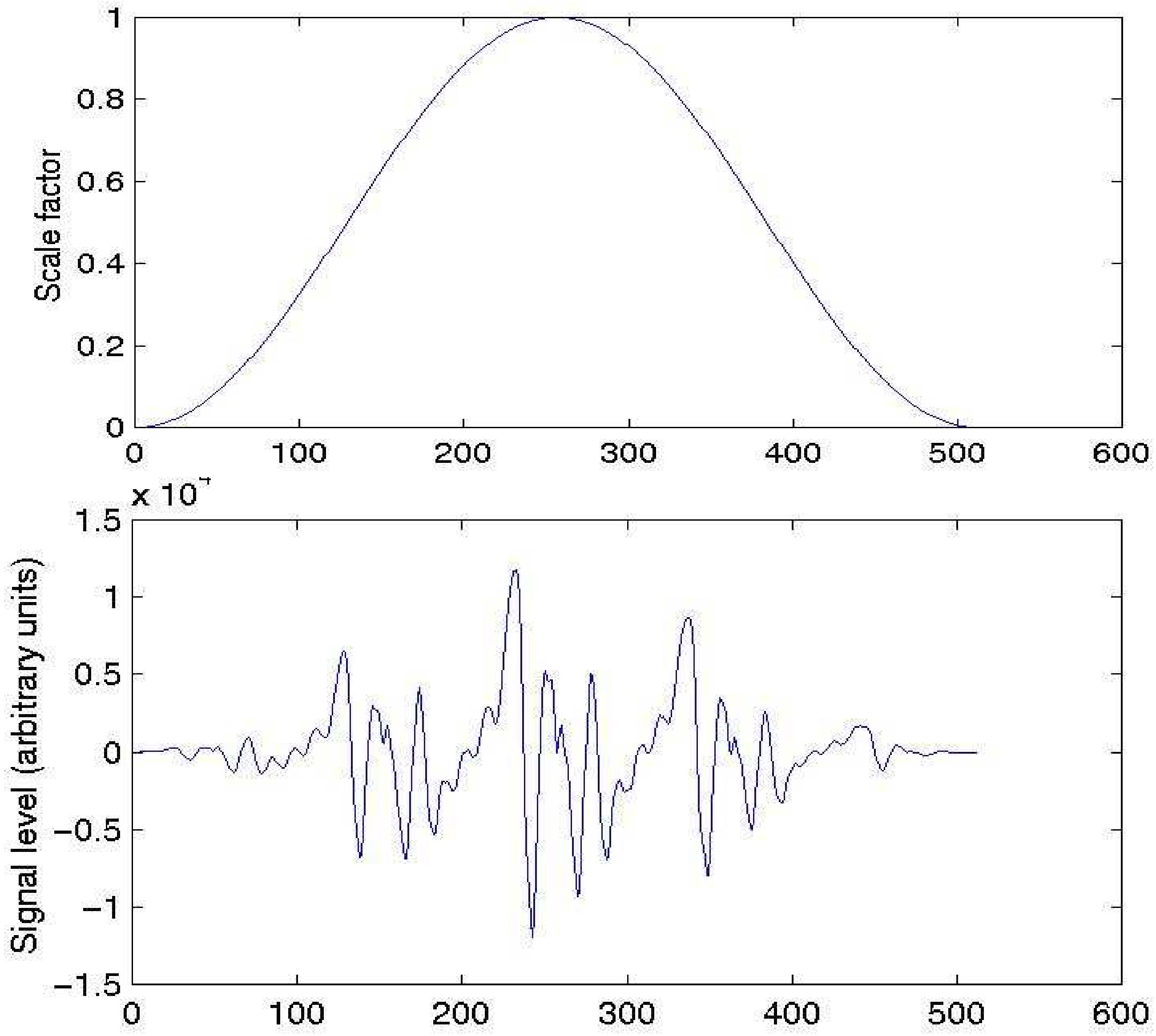
- $x[t] = -a_1\, x[t-1] - a_2\, x[t-2] - a_3\, x[t-3] \ldots - a_p\, x[t-p\,]$
  $+ e[t]$

Fourier (Spectral) Analysis

Jean Baptiste Joseph Fourier (1768-1830)

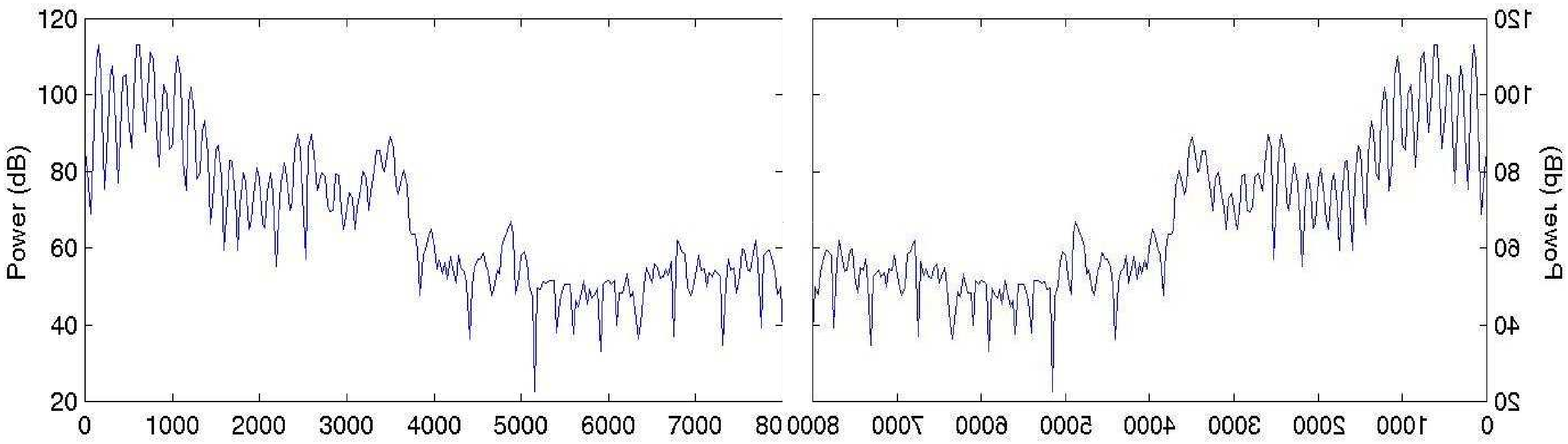


Windowing


Scale factor
Signal level (arbitrary units)
x 10
0
100
200
300
400
500
600
1
0.8
0.6
0.4
0.2
1.5
0.5
−0.5
−1
−1.5

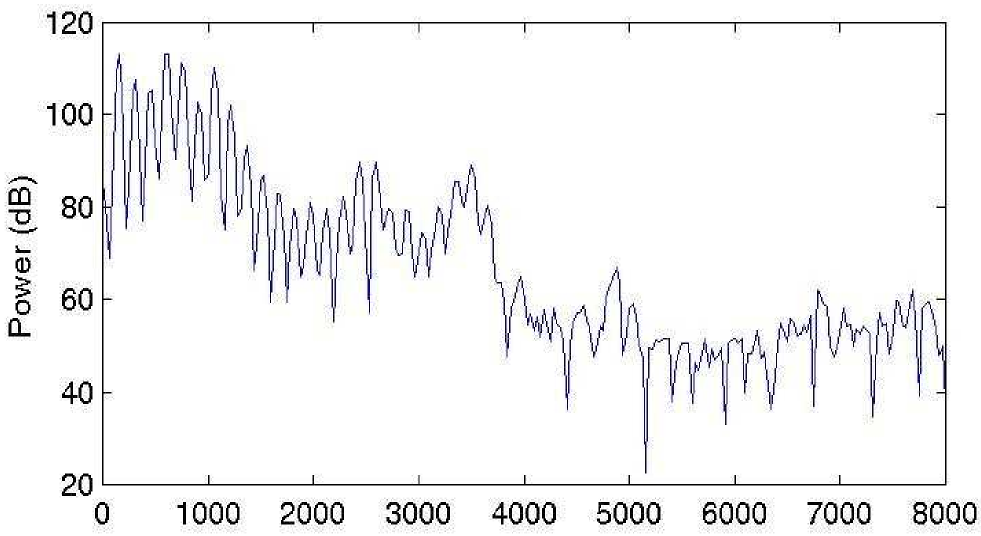
Fast Fourier Transform

- Cooley and Tukey, mid 1960's
- e.g. for Power Spectrum

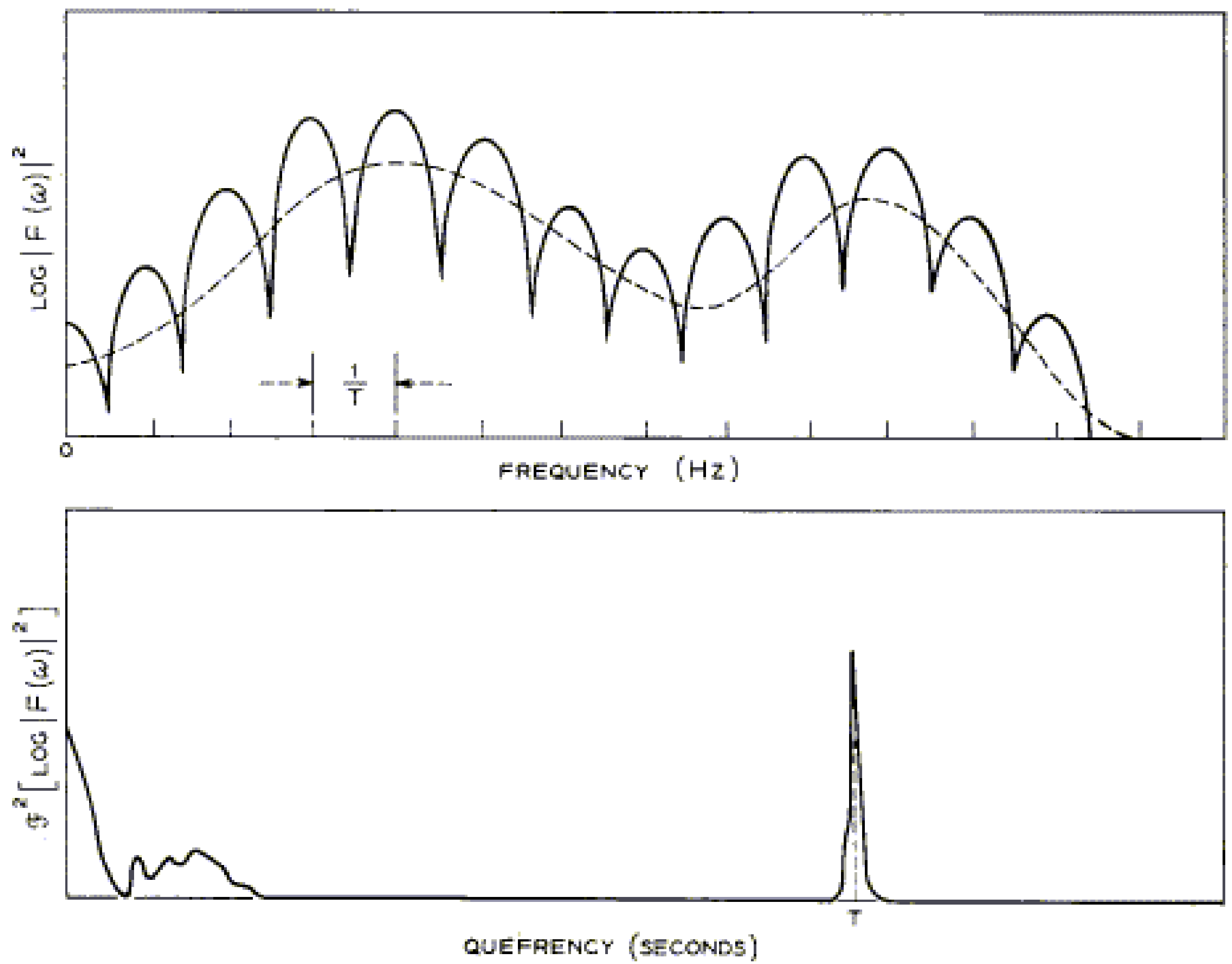


Fast Fourier Transform

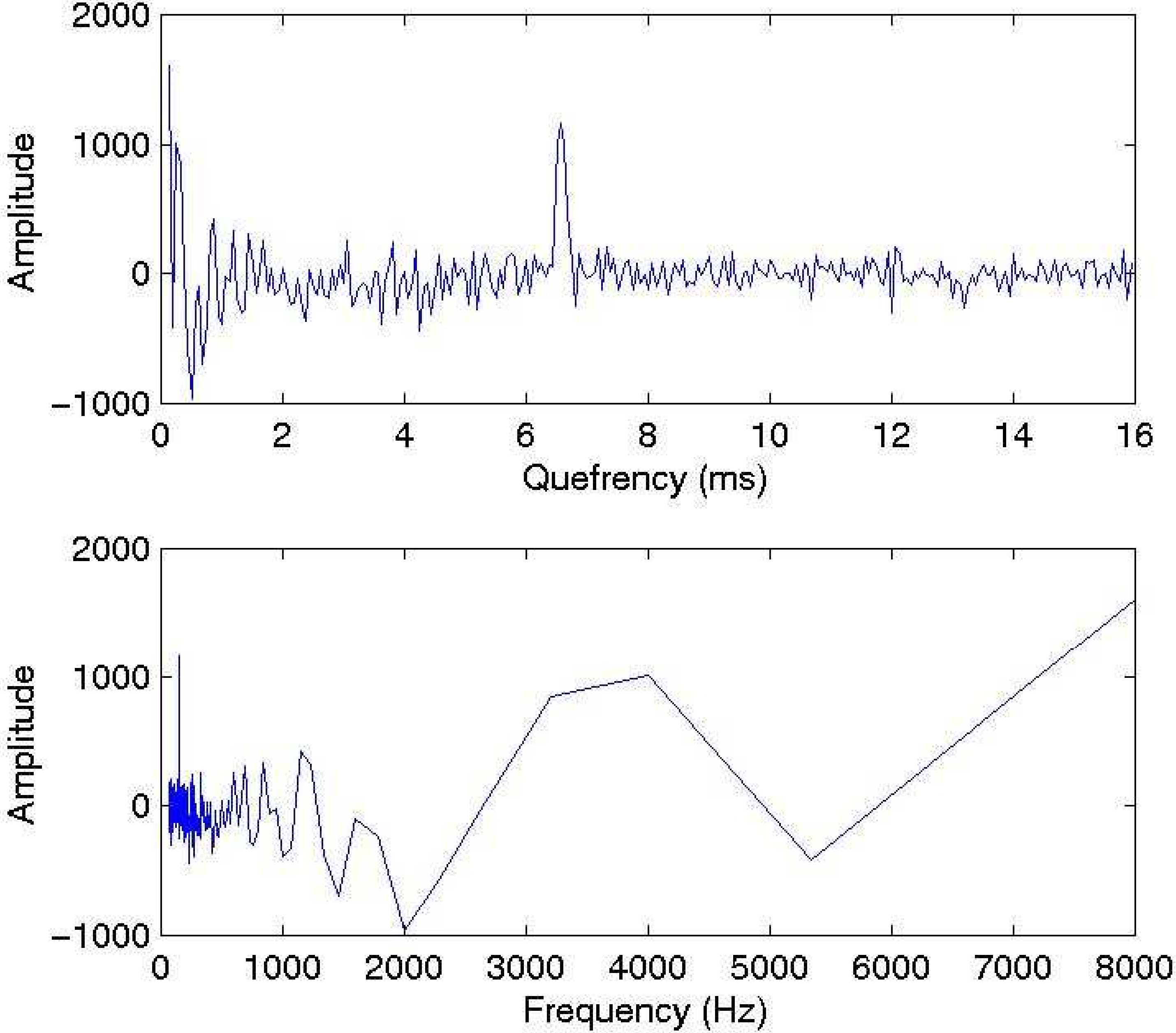
- Cooley and Tukey, mid 1960's
- e.g. for Power Spectrum



Cepstrum (Noll 1967)

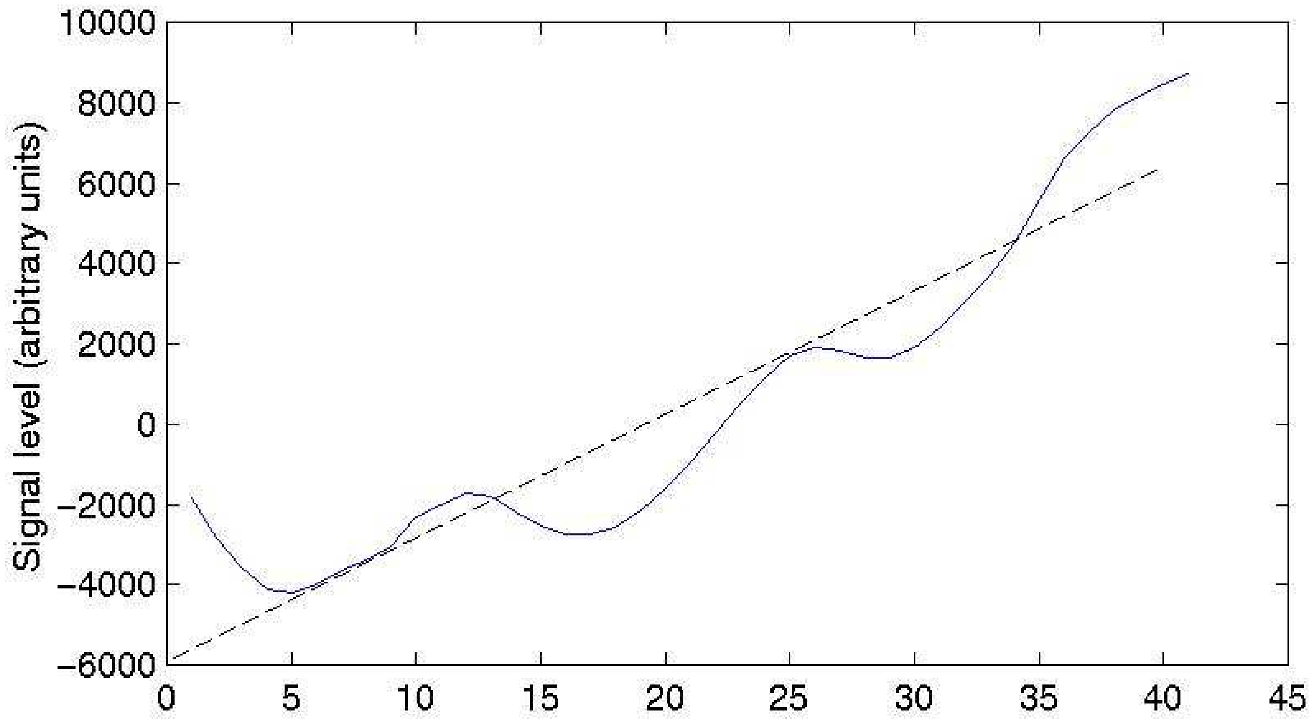


Cepstrum

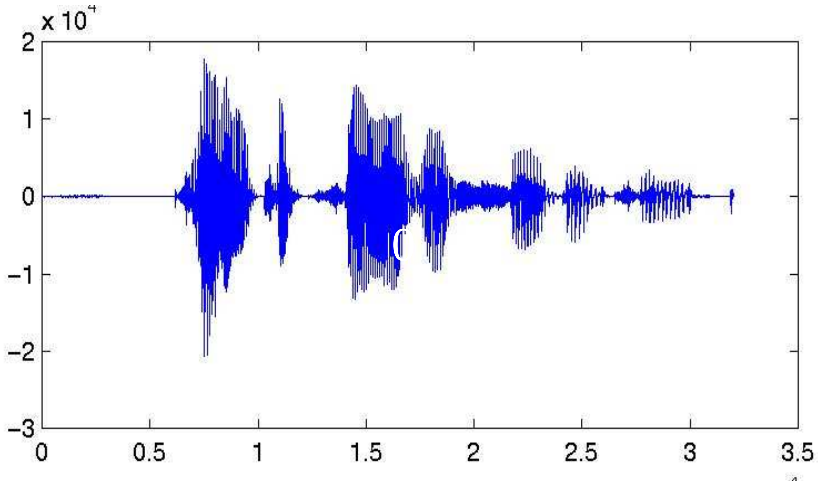

2000
1000
0
−1000
Amplitude
0
2
4
6
8
10
12
14
16
Quefrency (ms)
2000
1000
0
−1000
Amplitude
0
1000
2000
3000
4000
5000
6000
7000
8000
Frequency (Hz)

Linear Prediction of speech

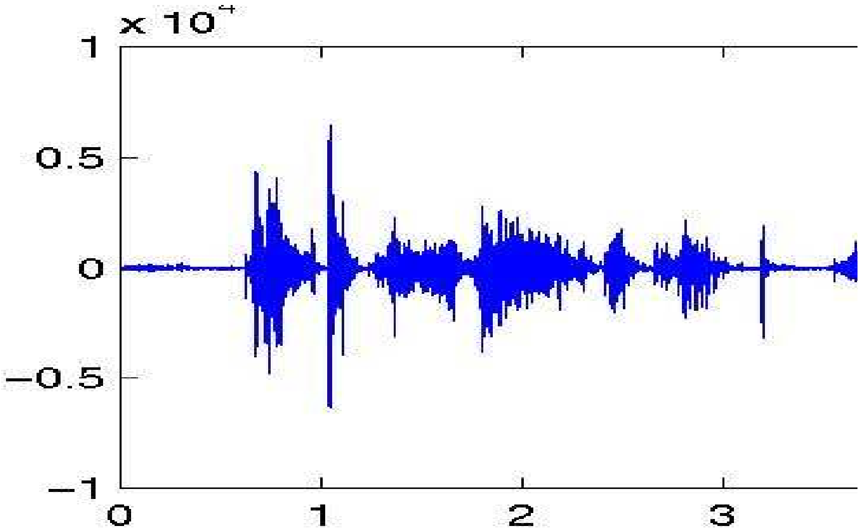
- We estimate the current sample as a linear combination of the previous p samples:
- $x[t] = -a_1\, x[t-1] - a_2\, x[t-2] - \ldots - a_p\, x[t-p] \qquad + e[t]$



Linear Prediction of speech

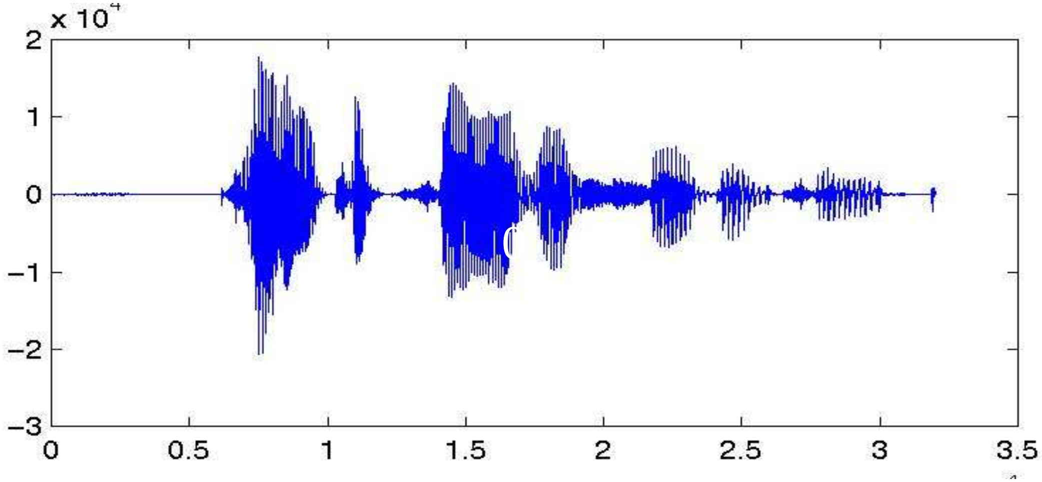


- Original

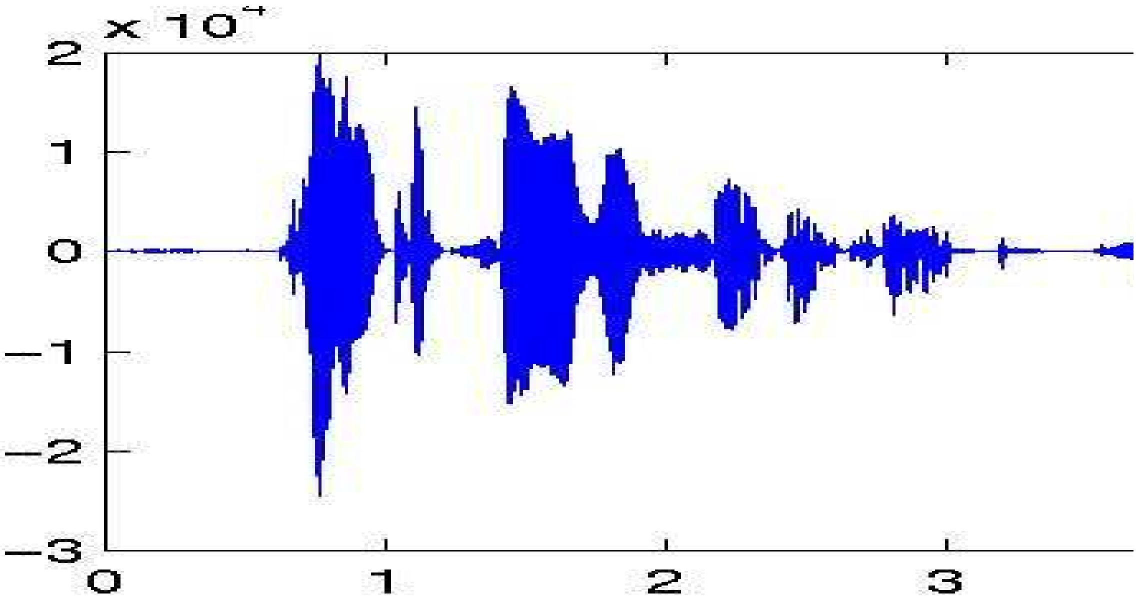


- Error

Linear Prediction of speech

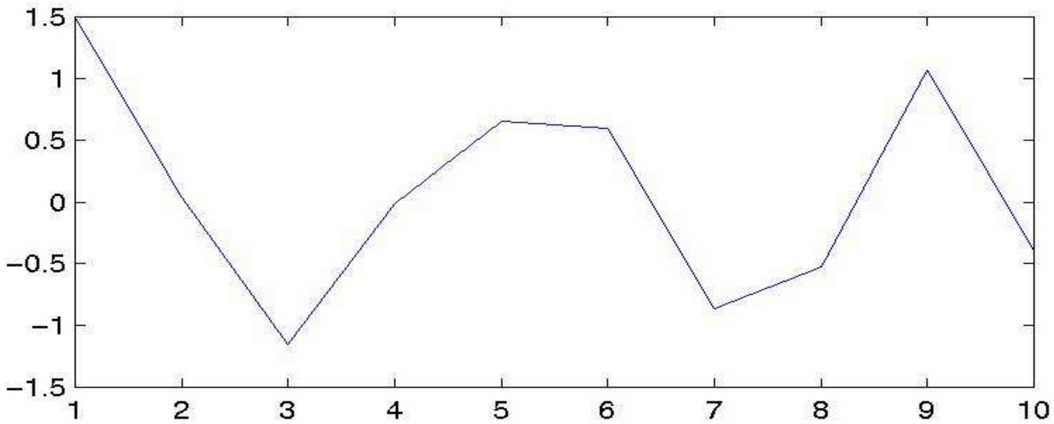


- Original

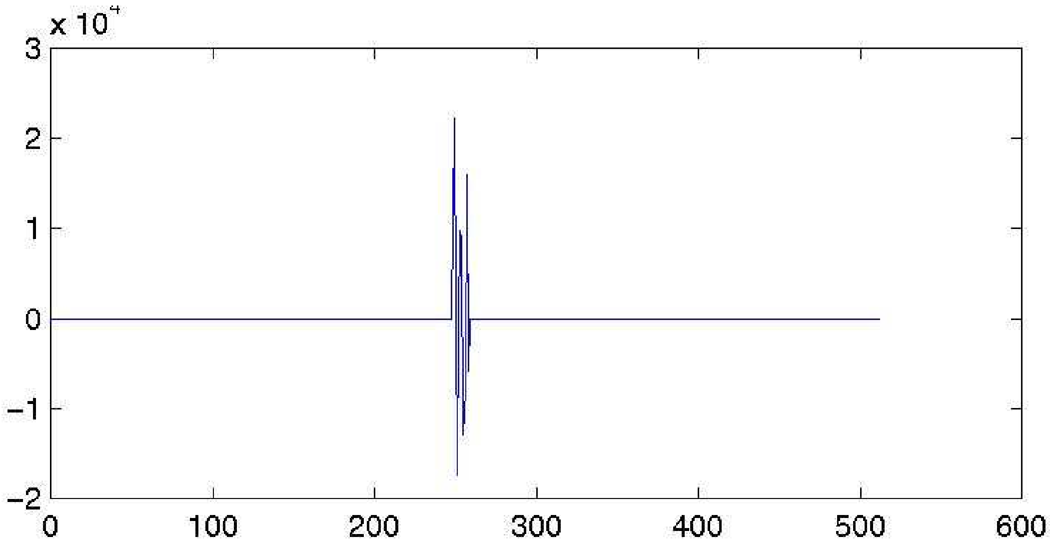


- Resynthesized excluding error

A magic trick



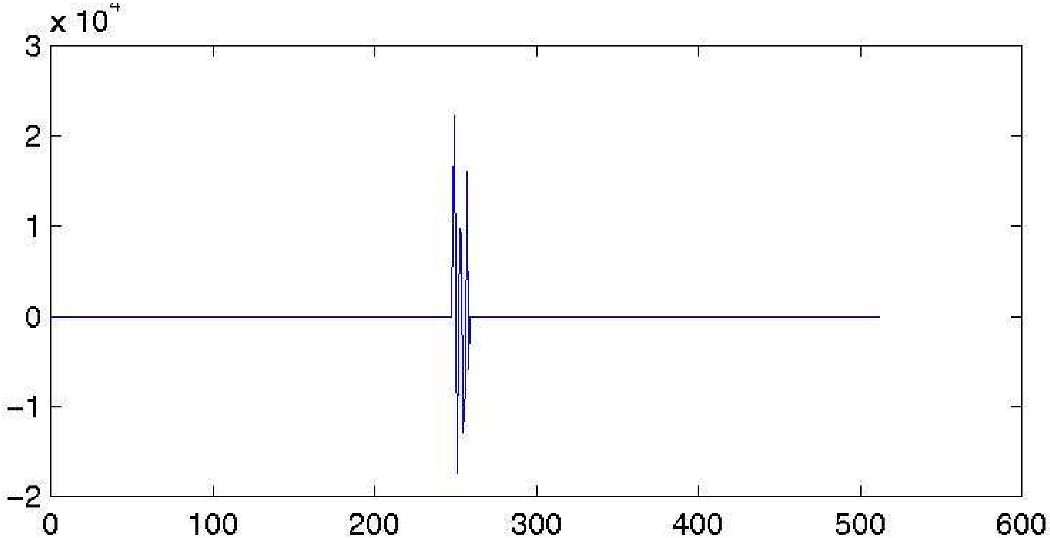
- 10 LPC coefficients from an [ɑ]



- Same 10 coeffs, scaled up and zero-padded to 512 samples long

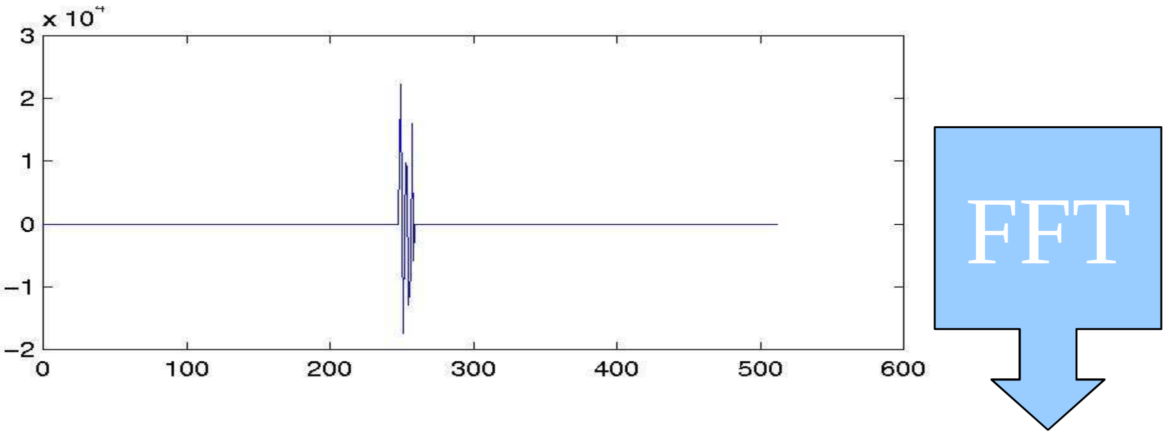
(251 each side)

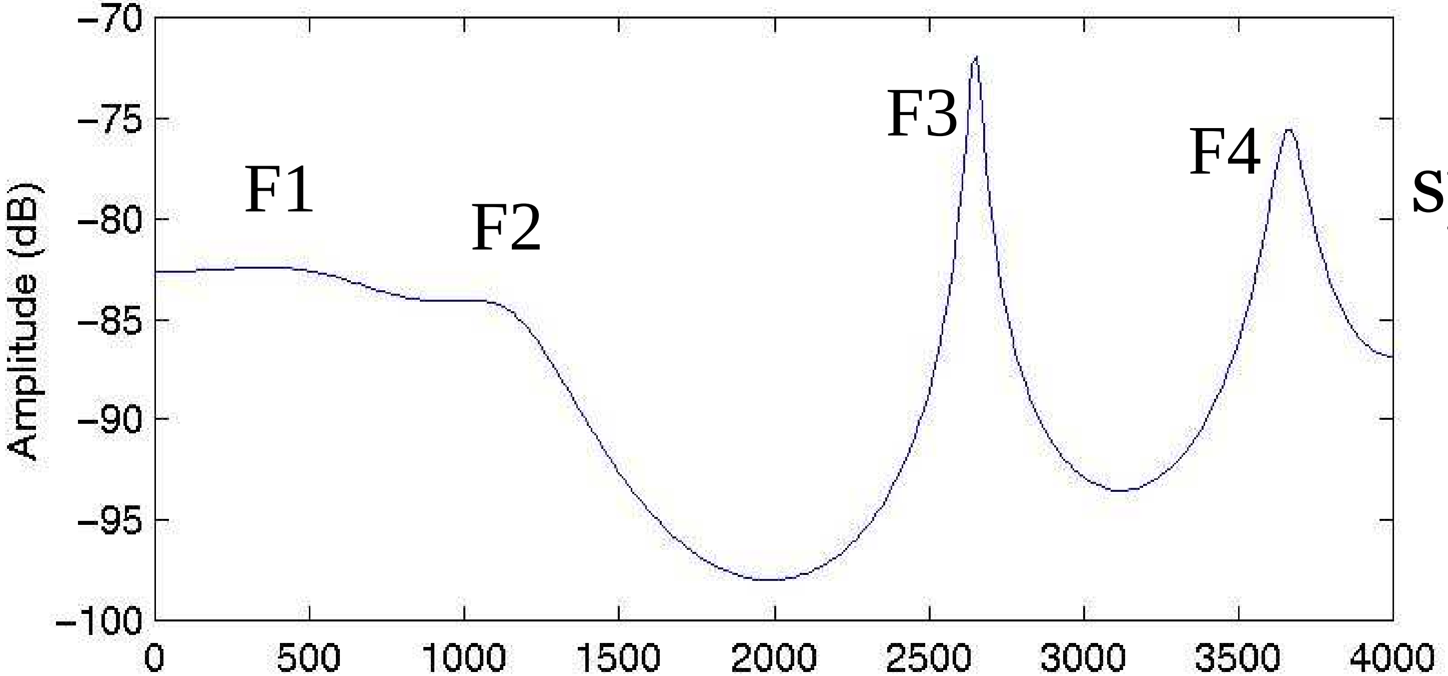
A magic trick



- Same 10 coeffs, scaled up and zero-padded to 512 samples long

A magic trick





LP spectrum

Shameless plug